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ANALYTICAL DETERMINATION OF TIME-DEPENDENT FUNCTIONS OF DRIFTS OF A TRIAXIAL GYROSTABILIZER†

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Equations which describe the time-dependence of the vector of the instantaneous velocity of rotation of a triaxial gyrostabilizer (TGS) platform located on Earth, with respect to the gyroscopical system of coordinates, are obtained within the framework of the precession theory at an arbitrary polynomial dependence of the rate of the drift on overloads. Based on the properties of the TGS and the details of its use we discuss the approximation of these equations from which the analytical time-dependent function of the drift rate of a gyrostabilized platform is found. The results obtained significantly simplify the procedure of solving the problems of identification of the TGS's parameters and estimating its current orientation relative to the specified system of coordinates in inertial navigation systems.

The precision of modern inertial navigation systems that have a gyrostabilized platform as the basic element depends, to the large extent, on the accuracy of evaluating the rate of the non-compensated drift of the platform (of its own drift) considered, as a rule, with respect to the axes of a system of coordinates attached to it.

The present developed identification algorithms of the drift rate are based on representing the drift rate components by certain time-dependent models. These models are described by polynomial or exponential expressions which are acceptable, with reasonable accuracy, for short time intervals of motion of the platform only. In such a case, the type of function describing the dependence of the drift rate on time is chosen in a heuristical way. Hence, it is required to revise it permanently, and this also causes an error increase when the mutual orientation of the Earth's trihedron and that of the platform alters because of the non-linear dependence of the platform drift rate on the projections of the accelerations onto its axes.

The present identification algorithms require significant amounts of computer resource. In this connection, an analytical determination of the time-dependent function of the drift rate of the TGS platform located on Earth is of interest with regard to the non-linear dependence of the drift rate on the accelerations, with the TGS in the state of forced stabilization.

For now there is a widely used function describing the vector of the drift rate in terms of overloads as a three-dimensional series which is a polynomial of the given degree with constant coefficients. This is due, in the first place, to the possibility of approximating a function derived theoretically, with the required accuracy and, in the second place, to the simplicity of finding the coefficients of such a polynomial in practice. In this connection we will take the three-dimensional polynomial with the specified coefficients as the drift rate function of overloads.

The process of TGS's functioning is characterized by the absence of great torques disturbing the platform and, in connection with this, it is possible to analyse its current orientation with the required accuracy within the framework of the precession theory. the angles of drift of modern TGSs in inertial space (but not relative to the astronomical trihedron attached to the Earth as assumed in existing models) are very small (they do not exceed fractions of a degree) during the long period of time (up to several hours). This enables one to write, using the known results [1], a system of equations which describe the time-dependence of the vector of the instantaneous drift rate of the stabilized platform in the following form

$$
\begin{gathered}
(\omega_X^{-1}\Lambda_1^{\cdot}) = (\omega_Y\omega_X^{-1})^{\cdot} + \omega_Z, \quad (\omega_Y^{-1}\Lambda_2^{\cdot})^{\cdot} = (\omega_X\omega_Y^{-1})^{\cdot} - \omega_Z \\
(\omega_Y^{-1}\Lambda_3^{\cdot}) = (\omega_Z\omega_Y^{-1})^{\cdot} + \omega_X, \quad (\Lambda_1, \Lambda_2, \Lambda_3)^{\mathrm{T}} = \Lambda \\
\Lambda = \mathbf{M}^{-1}\cdot\Omega - \mathbf{M}^{-1}\cdot\mathbf{U}, \quad \Omega = (\omega_X, \omega_Y, \omega_Z)^{\mathrm{T}}
\end{gathered}
\tag{1}
$$

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Here ω_X, ω_Y and ω_Z are the projections of the drift rate of the platform onto the axes of the gyroscopical trihedron, **M** and **U** are the known matrix-valued functions of time whose analytical expressions, along with deriving Eqs (1), have been given in [1], the overdot denotes differentiation with respect to time.

In order to find an analytical representation of a solution of the above equations we use the fact that the absolute variations of the platform drift rate remain small during the entire period of functioning (they do not exceed $10^{-7}\,\mathrm{s}^{-1}$ during 10 min of functioninng of real TGSs). This enables us, with a high degree of accuracy, to assume that the direction of the axis of its instantaneous rotation does not alter in the inertial system of coordinates over the specified time intervals.

Such an assumption is equivalent to the following conditions, which are to be satisfied by the projections of the drift rate during the specified time intervals

$$(\omega_X^2 + \omega_Y^2 + \omega_Z^2)^{1/2} = \nu, \quad \omega_j = c_j \nu, \quad c_j = \mathrm{const}, \qquad j = X, Y, Z. \tag{2}$$

Use of the relations specified in (1) leads to the system of three equations with respect to unknowns ν and c_j ($j = X, Y, Z$). Since the initial mutual orientation of the Earth and the platform is known, use of the conditions (2), at $t = 0$, enables us to find the values c_j for the first time interval of motion of the platform [1].

Having c_j we combine the original equations into the equation which describes the magnitude of the instantaneous velocity of rotation of the platform ν varying in time

$$\begin{gathered} ((a\nu + b)^{\cdot} \nu^{-1})^{\cdot} = c_X \nu, \quad a = (c_X^{-1}, c_Y^{-1}, c_Z^{-1})\mathbf{M}^{-1}(c_X, c_Y, c_Z)^{\mathrm{T}} \\ b = l_1 c_X^{-1} + (l_2 + l_3) c_Y^{-1}, \quad l_i = -(m_{i1}u_1 + m_{i2}u_2 + m_{i3}u_3) \end{gathered}$$

(m_{ij} and u_j are the corresponding elements of $\mathbf{M}^{-1}$ and **U**). Changing the variables $y = \ln \nu$ we write this equation in the form

$$y^{\cdot\cdot} + \delta_1 y^{\cdot} + \delta_2 y^{\cdot} e^{-y} + \delta_3 e^{-y} + \delta_4 e^{y} + \delta_5 = 0 \tag{3}$$

This is the initial equation for investigating the time evolution of the rate of the platform proper drift. We will next find the character of the time-dependent function of the drift rate based on the assumptions of magnitudes and values of variations in time of the coefficients and of the variables of Eq. (3) which follow from properties of the gyrostabilizer and features of its use. We assume that the coefficients of Eq. (3)

$$\delta_1 = \dot{a} a^{-1}, \quad \delta_2 = -\dot{b} a^{-1}, \quad \delta_3 = \ddot{b} a^{-1}, \quad \delta_4 = -c_X a^{-1}, \quad \delta_5 = \ddot{a} a$$

are constant within the time interval of motion of the platform, in which conditions (2) are satisfied.

According to the results of a simulation when there are no non-gravitational accelerations: $A_N = A_E = 0$, $A_L = -g$, we have $|\Delta\delta_i \delta_i^{-1}|_{\max} \leq 10^{-2}$, $j = X, Y, Z$; $i = 1, 2, \ldots, 5$ for real gyrostabilizers with $|\omega_j|_{\max} = (2\text{–}3) \times 10^{-6}\,\mathrm{s}^{-1}$

If we neglect in Eq. (3) the term with the coefficient $\dot{b} a^{-1}$ being very small for real gyrostabilizers (for instance, we have $|\dot{b} a^{-1}| \leq 10^{-4}$ for $t \in [0, 10^4]$ s with $|\omega_j|_{\max} \sim 10^{-6}\,\mathrm{s}^{-1}$) we obtain

$$y^{\cdot\cdot} + \dot{a} a^{-1} y^{\cdot} + \ddot{b} a^{-1} e^{-y} - c_X a^{-1} e^{y} + \ddot{a} a^{-1} = 0 \tag{4}$$

If we represent the sum of the exponential terms in the form

$$\begin{gathered} (c_X - \ddot{b})(a \operatorname{sh} q)^{-1} \operatorname{sh}(y - q) = i(\ddot{b} - c_X)(a \operatorname{sh} q)^{-1} \sin[i(y - q)] \\ q = \operatorname{arth}[(\ddot{b} - c_X)(\ddot{b} + c_X)^{-1}] \end{gathered}$$

and change the variables $z = y - q$ and $z_1 = i(y - q)$, we write Eq. (4) as follows:

$$\begin{gathered} \ddot{z}_1 + \gamma_2 \dot{z}_1 + \gamma_1 \sin z_1 + \gamma_0 = 0 \\ \gamma_0 = i(\ddot{q} + \dot{a} a^{-1} \dot{q} + \ddot{a} a^{-1}), \quad \gamma_1 = i(\ddot{b} - c_X)(a \operatorname{sh} q)^{-1}, \quad \gamma_2 = \dot{a} a^{-1} \end{gathered} \tag{5}$$

We neglect the components with the coefficients γ_0 and γ_2, because of their small values with respect to value $y^{\bullet\bullet}$. This is possible and advantageous in (5) for gyrostabilizers used in precision systems. In this case, we obtain

$$\ddot{z}_1 + i(\ddot{b} - c_X)(a \operatorname{sh} q)^{-1} \sin z_1 = 0 \tag{6}$$

i.e. the equation of oscillations of a pendulum whose solution is known. By making the inverse substitution of variables we write the expression for the instantaneous velocity of rotation

$$(i(b^{\cdot\cdot} - c_X)(a\,\mathrm{sh}\,q)^{-1})^{1/2}(t - t_0) = \int_{\chi_0}^{\chi} ((1 - u^2)(1 + k^2 - u^2))^{-1/2} du$$
$$\chi = ik^{-1}G, \quad \chi_0 = ik^{-1}G_0 \tag{7}$$
$$k^2 = G_0^2 + i(\nu_0^{\cdot}\nu_0^{-1} - q_0^{\cdot})^2 a\,\mathrm{sh}\,q(4(b^{\cdot\cdot} - c_X))^{-1}, \quad G = \mathrm{sh}\,[1/[(\ln \nu - q)]$$

The analytical representation of the velocity ν is not possible in this case. On the other hand, the magnitude of the coefficient $b^{\cdot\cdot}a^{-1}$ is very small ($\sim 10^{-8}$) in Eq. (4). This enables us to examine the equation

$$y^{\cdot\cdot} - c_X a^{-1} e^y = 0,$$

instead of Eq. (6).

The solution of this equation is known [2, p. 499]. Taking into account the possibility of changing the sign of the coefficient $c_x a^{-1} = c_0$ and making the substitution $y = \ln \nu$ we find the required expression

$$\nu = \begin{cases} c_1\,\mathrm{ch}^{-2}\,1/2W, & \text{for} \quad c_0 < 0, \quad c_1 > 0 \\ c_1\,\mathrm{sh}^{-2}\,1/2W & \text{for} \quad c_0 > 0, \quad c_1 > 0 \\ -c_1 \sin^2 i\,1/2W & \text{for} \quad c_0 > 0, \quad c_1 < 0 \end{cases} \tag{8}$$
$$W = c_1^{1/2}\beta(t - c_2), \quad \beta = (2|c_0|)^{1/2}$$
$$c_1 = ((\nu_0^{\cdot}\nu_0^{-1})^2 - 2\nu_0 c_0)(2|c_0|)^{-1}$$

(c_2 is the constant value specified from the solution (8) with $t = 0$).

The use of the different formulae, Eqs (7) and (8), for the magnitude of the drift rate ν, whose choice can be carried out in accordance with estimates of the corresponding coefficients of original Eq. (3), is possible for the different parts of motion of the gyrostabilizer.

The bound of the time interval in which the analytical expression of ν remains adequate can be specified by testing whether the conditions $\omega_j\omega_i^{-1} = \text{const}$, $i, j = X, Y, Z$ in which the computation of ω_i with the required accuracy is carried out according to [1], are satisfied.

The computation of values c_X, c_Y and c_Z fixed in the next interval of motion of the gyrostabilized platform is carried out by the usual recalculation of the coordinates with regard to the value of the angle of finite rotation of the previous interval $[t_0, t_0]$

$$\int_{t_0}^{t_k} \nu(s)\,ds$$

In order to verify if providing the required accuracy for the representation of the real drift rate of a TGS is possible, the numerical simulation of the evolution of the drift rate of gyrostabilizer's platform has been carried out using the approximations obtained from the solution of Eq. (3).

Current real values of projections of the drift rate in the inertial system of coordinates were compared with analogous values based on the analytical expressions obtained.

The peak deviation of values of the projections of the drift rate in the time interval (0–10^3) s did not exceed 1%.

We have reduced the time period of solving the problem using the approximation by a factor of 2.2 and the maximum capacity of the computer mainframe memory by a factor of 1.8.

Thus, the results obtained enable us to conclude that the practical use of the suggested approximation of the drift rate of the platform not only is possible but expedient from the standpoint of computer time.

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